## Lab 2: BINARY NUMBERS AND TRUTH TABLES

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SummaryThis lab introduces Truth Tables as graphic organizers for digital logic analysis. In thislab you will learn about binary numbers, truth table practice, and an application of truthtables.
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## Building on Lab 1

In the previous lab, we covered digital logic operators and gates including AND, OR, NOR, XOR, NAND, and MUX. If you recall, inputs and outputs can be represented as zeros and ones. This lab will allow you to check your intuition about the Digital Trainer's 10 Challenges with zeros and ones organized by a Truth Table.

OPTIONAL: If it is helpful for you, copy your guesses from Lab 1's Table 1 to this lab's Table 1 for easy access. Otherwise, have your answers from Lab 1 readily available to you throughout this lab.

Table 1: Digital Trainer Hypotheses

| Challenge | How many inputs? | What is the corresponding logic function or operator? |  |  |  |  |
| :---: | :--- | :---: | :--- | :--- | :--- | :--- |
| 1 |  | AND | OR | NAND | NOR | XOR |
| 2 |  | AND | OR | NAND | NOR | XOR |
| 3 |  | AND | OR | NAND | NOR | XOR |
| 4 |  | AND | OR | NAND | NOR | XOR |
| 5 |  | AND | OR | NAND | NOR | XOR |
| 6 |  | AND | OR | NAND | NOR | MUX |
| 7 |  | AND | OR | NAND | NOR | MUX |
| 8 |  | AND | OR | NAND | NOR | MUX |
| 9 |  | AND | OR | NAND | NOR | MUX |
| 10 |  |  |  |  |  |  |

## Binary Numbers

## Premise

Before we introduce Truth Tables, we need a basic understanding of binary numbers. Numbers are binary if they use "base-2" arithmetic. This means every digit in a number can only be a 0 or a 1 ; there are only two possibilities. In regular arithmetic, every digit in a number can be a 0,1 , $2,3,4,5,6,7,8$, or 9 . These 10 possibilities for each digit mean we use "base-10" arithmetic in everyday life. Electronics, however, are much more simplistic, so a one-digit number like 8 in base-10 is equivalent to 1000 in binary. How did this arise?

Let's start with base-10 numbers. Base-10 numbers are made up of digits. The rightmost digit is the "ones" place. Moving left, the place increases by a factor of 10 . Take 125 for example.


Figure 1: Base-10 Math
Figure 1 shows the arithmetic value of 125 is preserved if we view 125 as having three sub-locations (Oth blank, 1st blank, and 2nd blank). Knowing 125 is base-10, we can break down the number by its one's, ten's, and hundred's places. We can think of binary numbers in the same way. Take 1000 in Figure 2, and note how each digit can only be zero or one.

| NAME | "Eight's Place" | "Four's Place" | "Two's Place" | "One's Place" |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| EXAMPLE NUMBER | 1 | 0 | 0 | 0 | (base-2) |
| LOCATION | 3rd blank | 2nd blank | 1st blank | 0th <br> blank |  |
| BASE MEANING | $2^{3}$ | $2^{2}$ | $2^{1}$ | $2^{0}$ |  |
| ARITHMETIC | $\left(1 \times 2^{3}\right)+\left(0 \times 2^{2}\right)+\left(0 \times 10^{1}\right)+\left(0 \times 10^{0}\right)=8$ in base-10 |  |  |  |  |

Figure 2: Binary (Base-2) Math
We can see that base-10 and binary numbers can be understood similarly. Binary digits are incremented by powers of two, whereas base-10 digits are incremented by powers of ten.

For the purposes of this lab, you should be able to recognize and write out small binary numbers equivalent to $0-15$ in base-10. To get you started, Table 2 lists the binary equivalents of the numbers 0-9.

Table 2: Base-10 and Base-2 Numbers

| BASE-10 | BASE-2 (BINARY) |
| :--- | :--- |
| 0 | 0 |
| 1 | 1 |
| 2 | 10 |
| 3 | 11 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |
| 8 | 1000 |
| 9 | 1001 |

## Challenge Questions

To test your understanding, find the binary equivalents of the numbers 11, 12, and 15. Hint: find the largest powers of 2 which fit in the number.

- 11 =
- $12=$
- $15=$

Find the base-10 equivalents of these numbers. Hint: each digit corresponds to a power of 2.

- $1010=$
- $1101=$
- $1110=$


## What is a Truth Table?

Truth Tables are graphic organizers for keeping track of patterns between the inputs and outputs. Remember that " 1 " represents "on" and " 0 " represents "off."

The general form is a T-chart with inputs on the left and outputs on the right. The labels are located at the top of the T-chart, specifying the name of input and output signals. A blank Truth Table with two input signals, $A$ and $B$, and one output signal $C$ is depicted in Figure 3.


Figure 3: A Blank Truth Table
The information in a Truth Table represents whether the signal was true/on (shown by a 1 ) or false/off (shown by a 0). Given x inputs, there are $2^{x}$ entries of outputs in the Truth Table. The Truth Table inputs correspond to binary numbers, with 0 at the top and $\left(2^{x}-1\right)$ at the bottom. Figure 4 shows a color-labeled diagram of each column and row in a Truth Table with sample data.


Figure 4: A Truth Table with Data
Notice that, because we have two inputs, there are $2^{2}=4$ possible combinations of zeros and ones. Counting in binary, that means the left side of the Truth Table goes from 00 to 11 ( 0 to 3 in base-10). The right side represents the result of each possibility. In this case, the output was only true when both input signals were on. All other cases -- A and B both off (00), A off and B on (01), A on and B off (10) -- resulted in false outputs.

## Application

Figures 5a-b have 10 blank Truth Tables, 1 for each of the 10 challenges you observed in Lab 1 . For the best learning experience, do not look at Figure 6 until you complete Figures 5a-b.

- For each row (an input combination), record the output as a 1 (for on) or 0 (for off).
- Use the output information to guess/verify what the corresponding operator is: AND, OR, NAND, NOR, XOR, and MUX.
- Refer back to the Digital Trainer activity in LabsLand as needed. Also, refer to the definitions for these operators in Lab 1. Your guess can be the same or different from your guess in Lab 1.
- Note that some of the Truth Tables have the input signals filled in as hints; for the others, you must complete all the columns.


## Challenge 1

Operator:

| SW1 | SW0 | LEDout |
| :--- | :--- | :--- |
| 0 | 0 |  |
| 0 | 1 |  |
| 1 | 0 |  |
| 1 | 1 |  |

## Challenge 3

Operator:

| SW1 | SW0 | LEDout |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |

## Challenge 2

Operator:

| SW1 | SW0 | LEDout |
| :--- | :--- | :--- |
| 0 | 0 |  |
| 0 | 1 |  |
| 1 | 0 |  |
| 1 | 1 |  |

## Challenge 4

Operator:

| SW1 | SW0 | LEDout |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |

Figure 5a: Digital Trainer Blank Truth Tables

Challenge 5
Operator:

| SW1 | SW0 | LEDout |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |

Challenge 6
Operator:

| SW2 | SW1 | SW0 | LEDout |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 |  |
| 0 | 0 | 1 |  |
| 0 | 1 | 0 |  |
| 0 | 1 | 1 |  |
| 1 | 0 | 0 |  |
| 1 | 0 | 1 |  |
| 1 | 1 | 0 |  |
| 1 | 1 | 1 |  |

## Challenge 9

Operator:


Challenge 7
Operator:

| SW2 | SW1 | SWO | LEDout |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 |  |
| 0 | 0 | 1 |  |
| 0 | 1 | 0 |  |
| 0 | 1 | 1 |  |
| 1 | 0 | 0 |  |
| 1 | 0 | 1 |  |
| 1 | 1 | 0 |  |
| 1 | 1 | 1 |  |

Challenge 10
Operator:


Figure 5b: Digital Trainer Blank Truth Tables

## Recognizing Patterns

Figure 4 illustrates 10 Truth Tables for the 10 challenges you observed in LabsLand. For each Truth Table, the corresponding operator for the challenge is given.
Challenge 1: NOR

| SW1 | SW0 | LEDout |
| :--- | :--- | :--- |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

Challenge 2: AND

| SW1 | SW0 | LEDout |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Challenge 3: XOR

| SW1 | SW0 | LEDout |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Challenge 4: NAND

| SW1 | SW0 | LEDout |
| :--- | :--- | :--- |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Challenge 5: OR

| SW1 | SW0 | LEDout |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Challenge 6: MuX

| SW2 | SW1 | SW0 | LEDout |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Challenge 7: OR

| SW2 | SW1 | SW0 | LEDout |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

Challenge 8 : NAND

| SW2 | SW1 | SW0 | LEDout |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Challenge 9: NOR

| SW2 | SW1 | SW0 | LEDout |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

Challenge 10: AND

| SW2 | SW1 | SW0 | LEDout |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

Figure 6: Digital Trainer Truth Tables

Compare your Truth Tables in Figures 5a-5b the Truth Tables in Figure 6. If your guess matches, compare your initial reasoning (before learning about Truth Tables) to the Truth Table process. If the tables do not match, revisit LabsLand and double check the system behavior. Describe what you observe in Table 3.

Table 3: Truth Table Reasoning

| Challenge | Guess | Did your guess match the operator? Why or why not? (1-2 sentences) |
| :---: | :--- | :--- |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |


| 6 |  |  |
| :--- | :--- | :--- |
| 7 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |

The reflection and Truth Tables you completed will be helpful in our transition to Boolean Algebra and K-Maps in the following lab. Understanding Truth Tables as graphic organizers will help you derive and represent relationships from observed I/O.

## Reflection and Observations

Lab 2 introduced you to Truth Tables and binary numbers. Reflect on the new topics and the things you found interesting and/or challenging in the space below.

What questions do you still have, if any?

